Physics 200 Name KEY

Exam 3 – Rotation and Gravity

60 pts November 24, 2008

This is a closed book examination. You may use a small 3x5 card with equations on it. There is extra scratch paper available. Explanations must be included with all answers – even multiple-choice questions. Your explanation is worth 75% of the possible points.

A general reminder about problem solving:

- Focus
  - Draw a picture of the problem
  - What is the question? What do you want to know?
  - List known and unknown quantities
  - List assumptions
- Physics
  - Determine approach – What physics principles will you use?
  - Pick a coordinate system
  - Simplify picture to a schematic (if needed)
- Plan
  - Divide problem into sub-problems
  - Modify schematic and coordinate system (if needed)
  - Write general equations
- Execute
  - Write equations with variables
  - Do you have sufficient equations to determine your unknowns?
  - Simplify and solve
- Evaluate
  - Check units
  - Why is answer reasonable?
  - Check limiting cases!
- Show all work!

The next two questions involve objects that are moving at a tangent to the normal above a planetoid. Both objects have identical velocities but object one is at a radial distance ½ of object two, $r_1=\frac{1}{2}r_2$.

1. [4 PTS] How are the two angular velocities related?

A. $\omega_1 = 4\omega_2$

[✓ B.] $\omega_1 = 2\omega_2$

C. $\omega_1 = \omega_2$

D. $\omega_1 = \frac{1}{2}\omega_2$

E. $\omega_1 = \frac{1}{4}\omega_2$

$V = \omega r$

$V_1 = \omega_1 r_1 \quad V_2 = \omega_2 r_2 \quad V_1 = V_2$

$\omega_1 r_1 = \omega_2 r_2 = \omega_2 2 r_1$

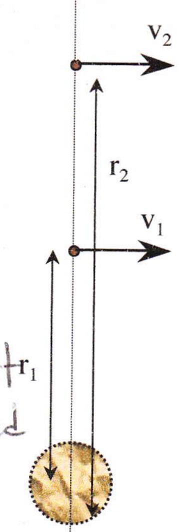
$\omega_1 = 2\omega_2$

2. [8 PTS] The more distant object has a velocity that allows it to stay in a stable orbit.

[✓ A.] Object one is traveling too slowly for a stable orbit.

B. Object one also is in a stable orbit.

C. Object one is traveling too fast for a stable orbit.



$\sum F = ma$

$\frac{GMm}{r^2} = m\frac{v^2}{r}$

stable orbital velocity for v_1 that is needed

$V_{orb} = \left(\frac{GM}{r_1}\right)^{1/2}$

$V_2 = \left(\frac{GM}{r_2}\right)^{1/2}$ this is a stable velocity

$V_1 = V_2 = \left(\frac{GM}{r_2}\right)^{1/2} = \left(\frac{GM}{2r_1}\right)^{1/2} = \left(\frac{GM}{r_1}\right)^{1/2}\frac{1}{\sqrt{2}}$

$V_1 = \frac{V_{orb}}{\sqrt{2}}$ too slow

3. [4 PTS] Two large heavy objects sit 0.2 m apart. The first object is 4 times more massive than the second mass, $m_1 = 4m_2$. The force of the first object on the second object is -2.0 N. The force of the second object on the first object is

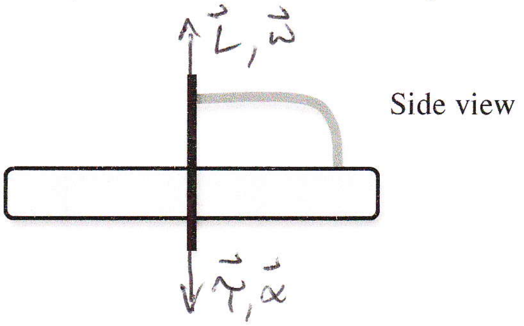
   A. 16 N
   B. 4.0 N
   [✓ C.] 2.0 N
   D. 1.0 N
   E. 0.5 N

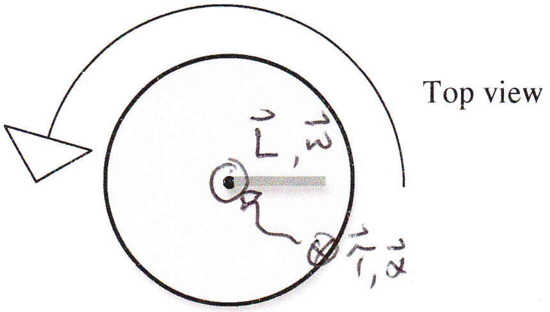
$\vec{F}_G = \frac{G m_1 m_2}{r_{12}^2}\hat{r}_{12}$

"For every ~~force~~ reaction there is an equal but opposite reaction"

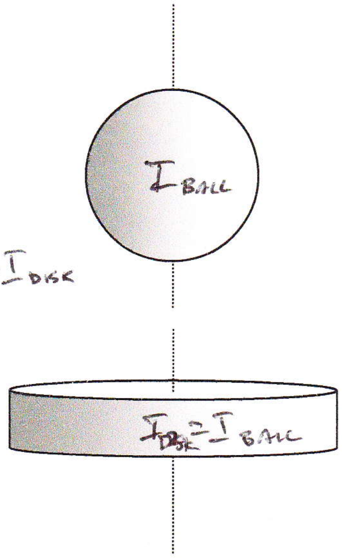
You can see from above that the two forces are equal magnitude but opposite direction

4. [4 PTS] You are on a playground merry-go-round (a spinning disk) that you started spinning in a counterclockwise direction as seen from the top (see arrow). There is a large frictional torque so you will have a short ride. Clearly draw and label (both on top and side views) the rotational vectors ($\vec{L}$, $\vec{\omega}$, $\vec{\alpha}$, $\vec{\tau}$) for ~~when~~ right after you have jumped on the merry-go-round.





You decide to spin a hollow ball and a solid disk (see picture below). Both the disk and ball have the same moment of inertia but the disk has a radius that is twice the ball's radius. You apply the same tangential force to the edge of each object for 2 seconds. The next two questions involve this ball and disk. NOTE: REMEMBER TO PROVIDE EXPLANATIONS.



5. [4 PTS] After being spun which object has more energy?
   A. The ball has more energy.
   B. Both the ball and disk have the same energy.
   [✓ C.] The disk has more energy.

$\Delta KE = \int \vec{F}\cdot d\vec{x} = \int \vec{\tau}\cdot d\vec{\theta}$

$KE = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{L^2}{I}$ since $L_{BALL} < L_{DISK}$ and $I_{BALL} = I_{DISK}$

$KE_{BALL} < KE_{DISK}$

6. [4 PTS] After being spun which object has a greater angular momentum?
   A. The ball has a greater angular momentum.
   B. Both the ball and disk have the same angular momentum.
   [✓ C.] The disk has a greater angular momentum.

$\int \vec{\tau}\cdot dt = \Delta\vec{L}$

$\tau = r \times F$

$\tau_{ball} = r_b F$ $\tau_{disk} = r_d F$ $r_d = 2r_b$

$\Delta L_{disk} = 2\Delta L_{ball}$

$\tau_{ball} = \frac{1}{2}\tau_{disk}$

7. [4 PTS] You have built a device capable of launching an object ($m_1 = 400$ kg) with energy (E_1) so that the object can just barely escape the earth's gravitational potential. You want to launch a smaller object ($m_2 = 200$ kg). What is the minimum energy required to launch this smaller object?

- A. $E_2 = 4E_1$
- B. $E_2 = 2E_1$
- C. $E_2 = E_1$
- [✓ D.] $E_2 = \frac{1}{2}E_1$
- E. $E_2 = \frac{1}{4}E_1$

$V_{esc} = \left(\frac{2GM_E}{R_E}\right)^{1/2}$

$-\frac{GM_E m}{R_E} + \frac{1}{2} m v_{esc}^2 = 0$

v

↖ Does not change

$KE = \frac{1}{2} m V_{esc}^2$ so m↓ KE↓

8. [4 PTS] Two solid balls have the same <u>mass</u>, $m_1 = m_2$ but different diameters, $r_1 = 2r_2$. Their moments of inertia are related such that

- [✓ A.] $I_1 = 4I_2$
- B. $I_1 = 2I_2$
- C. $I_1 = I_2$
- D. $I_1 = \frac{1}{2}I_2$
- E. $I_1 = \frac{1}{4}I_2$

$I \propto r^2$ $I \propto m$

so $\frac{I_1}{I_2} = \frac{r_1^2}{r_2^2} = \frac{(2r_2)^2}{r_2^2} = 4$ $I_1 = 4I_2$

9. [12 PTS] You are part of a NASA team that has identified a new planet VUWCHPHRADS-2009. This "near earth" planet has twice the radius and twice the mass of earth. The NASA team has asked you to

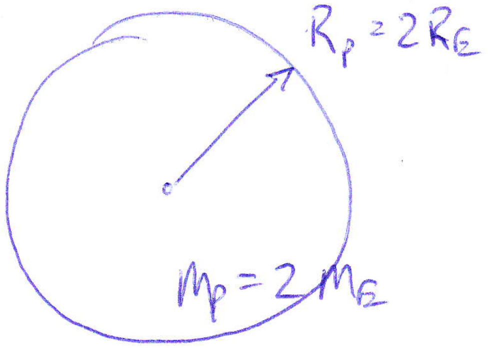
- A. Determine the gravitational acceleration you would feel on the surface of this planet. Express your answer in terms of "g_{earth}".
- B. Determine the escape velocity to launch an object from the surface of this planet. Compare to the escape velocity for an object on the surface of the earth.

10. [12 PTS] How to win when playing "Wheel of Fortune". You are spinning a solid disk (r = 1.2 m and m = 75 kg) that is free to rotate horizontally. The disk has a frictional torque of 60 Nm.

- A. How fast does the disk need to be rotating initially to make 2.25 rotations?
- B. How much energy does the disk have initially? (i.e. when it has maximum angular momentum)
- C. What is the minimum force you need to apply to the edge of the disk to achieve the desired angular velocity if you can only push on the disk for 0.2 meters?

Useful Data:
Mass of Earth = 6×10^{24} kg
Radius of the Earth = 6.4×10^{6} m
G = 6.67×10^{-11} Nm^2/kg^2

$\boxed{9}$



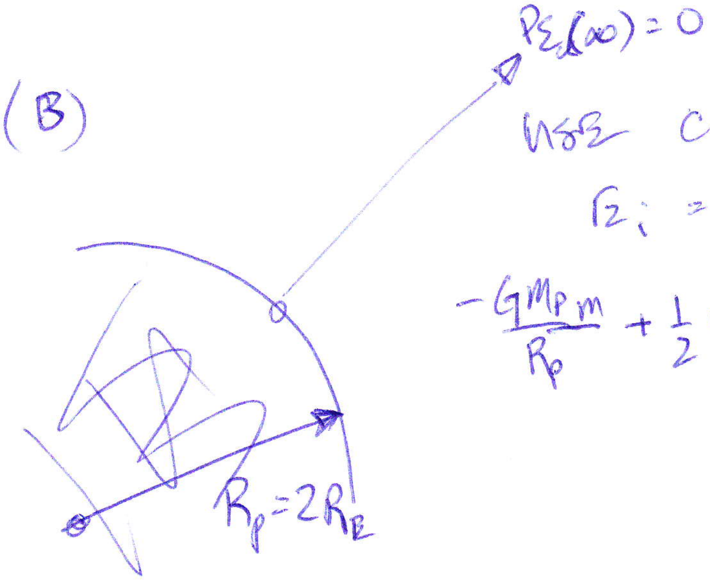
Gravitational force for an object on the surface of this planet

$$F_p = G\frac{M_p m}{R_p^2} = g_p m$$

(A) $$g_p = \frac{GM_p}{R_p^2} = \frac{G(2M_E)}{(2R_E)^2} = \frac{1}{2}\frac{GM_E}{R_E^2} = \frac{1}{2}g$$

so the gravitational acceleration on the surface of the planet is $\frac{1}{2}g = \frac{1}{2}9.8\, m/s^2 = 4.9\, m/s^2$

(B)



use conservation of energy

$$E_i = E_f$$

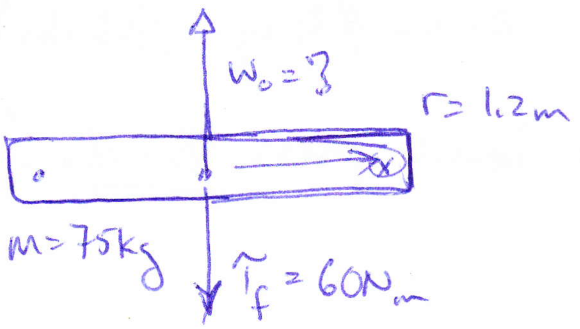
$$-\frac{GM_p m}{R_p} + \frac{1}{2} m V_{esc}^2 = 0$$

$$\underset{planet}{V_{esc}} = \left(\frac{GM_p 2}{R_p}\right)^{1/2} = \left(\frac{2G(M_E\cdot\cancel{2})}{\cancel{2}R_E}\right)^{1/2}$$

$$\underset{planet}{V_{esc}} = \underset{earth}{V_{esc}} = \left(\frac{2GM_E}{R_E}\right)^{1/2}$$

$$= 1.1\times 10^4\, m/s$$

$\boxed{10}$



$I = \int r^2 dm = \frac{1}{2} mr^2$ ← for a disk

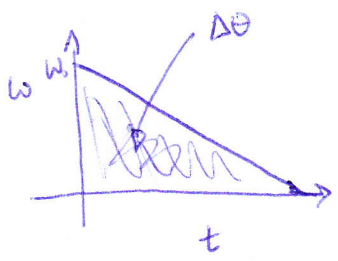
(A) WANT 2.25 rotations $\longrightarrow$ $\Delta\theta = \overset{\text{rad/rotation}}{(2\pi)}\, 2.25 = 4.5\pi$

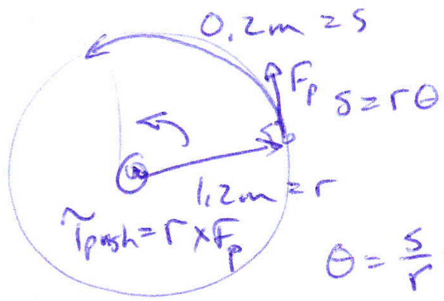
$\sum \tau = I\alpha \quad \alpha = \frac{-\tau_f}{I}$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega^2 - \omega_0^2 = 2\alpha\Delta\theta$





$\theta = \frac{s}{r} = \frac{0.2m}{1.2m} = 0.167\,rad$

$\omega = \emptyset \quad \alpha = \frac{-\tau_f}{\frac{1}{2}mr^2} \quad \Delta\theta = 4.5\pi$

$\omega_0 = \left[\frac{4\tau_f(4.5\pi)}{mr^2}\right]^{1/2}$

$\boxed{\omega_0 = 5.6\ rad/sec}$

units ✓
$m\uparrow \quad \omega_0\downarrow$ (more inertia)
$\tau_f\uparrow \quad \omega_0\uparrow$
$\Delta\theta\uparrow \quad \omega_0\uparrow$

(B) WANT INITIAL ENERGY (right after push)

$KE_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}\cancel{mr^2}\right)\frac{\cancel{4}\tau_f(4.5\pi)}{\cancel{mr^2}} = \tau_f(4.5\pi) \boxed{= 848\,J}$

$\cancel{KE_f}^{0} - KE_i = \Delta KE = -\int \tau \cdot d\theta = -\tau_f \cdot \Delta\theta = -\tau_f(4.5\pi) = -848\,J = -KE_i$

$KE_{rot\ initial} = 848\,J$

(C) $\sum \vec{\tau} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$ better to use energy...

$\Delta KE = \int \vec{\tau} \cdot d\theta \quad \tau_{net} = \tau_{push} - \tau_{fr}$

$\frac{\overset{\Delta KE}{848\,J}}{\underset{\Delta\theta}{0.167\,rad}} = \overset{\tau_{net}}{\tau_{push} - 60Nm}$

$KE_i = 0$
$KE_f = 848J$

~~5149~~ Nm $= \tau_{push} = 5150\,Nm$

$F_{push} = \frac{\tau_{push}}{r} = \frac{5150\,Nm}{1.2m} \boxed{= 4290\,N}$

Assume push is at right angle – F_{push} is quite large